

A Beamforming Algorithm for MIMO SWIPT Systems

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Abstract— Efficient usage of energy resources is a growing concern in today's communication systems. Energy harvesting is a new paradigm and allows the nodes to recharge their batteries from the environment. In this paper, we focus on the design of optimal linear beamformer for multi- input multi-output (MIMO) simultaneous wireless information and power transfer (SWIPT) system. We formulate the problem of maximizing the information rate while keeping the energy harvested at the energy receivers above given levels. Finally, simulation results demonstrate the efficiency of the proposed algorithm.

Keywords— simultaneous wireless information and power transfer (SWIPT), beamforming, MIMO (multi- input multi-output), minimum mean square error (MMSE), optimization.

I. INTRODUCTION

Recently, the exponential growth in the demand for high data rates in wireless communication networks has led to a tremendous need for energy. The rapidly escalated energy consumption not only increases the operating cost of communication systems, but also raises serious environmental concerns. As a result, green radio communications have received much attention in both academia and industry [1]-[5]. Simultaneous wireless information and power transfer (SWIPT) refers to using the same emitted electromagnetic (EM) wave-field to transport both energy that is harvested at the receiver, and information that is decoded by the receiver [6].

With SWIPT, one and the same wave-field is used to transmit energy and information. This has several advantages. First, separate transmission of power and information by time division is suboptimal in terms of efficiently using the available power and bandwidth. SWIPT, by contrast, may exploit integrated transceiver designs. Second, with SWIPT, interference to the communication systems can be kept under control. This is especially important in multi-user systems with many potential receivers who would suffer from interference [6].

With the aid of this promising technique, the transmitter can transfer power to terminals who need to harvest energy to charge their devices, which is especially important for energy-constrained wireless networks. Beamforming is another

promising technique which exploits channel state information (CSI) at the transmitter for information transmission [7], [8]. In wireless networks with simultaneous transmission of power and information, beamforming is anticipated to play an important role as well [7].

In this paper, we proposed a generalized framework for precoder and post-coder designs in MIMO (multi input multi output) SWIPT systems using the maximum information rate and MMSE (minimum mean square error) criterion.

The remaining parts of this paper are organized as follows. Section II delineates the system model and the problem formulation is presented. Section III presents our proposed algorithm to find the solutions to the problems using convex optimization. Simulation results are given in Section IV. Finally, Section V provides some concluding remarks of this research work.

Notations: $\mathbf{X} \in \mathbb{C}^{r \times c}$ denote the complex matrix with r rows and c columns, \mathbf{X}^H is the transpose and conjugate transpose (Hermitian operator) of the matrix \mathbf{X} . $|\mathbf{X}|$ is the determinant of the matrix \mathbf{X} . $E(\cdot)$ stands for the expectation operator. $\text{rank}(\mathbf{X})$ and $\text{trace}(\mathbf{X})$ denote the rank and trace operator of the matrix \mathbf{X} , respectively. \mathbf{I}_y is the identity matrix having y rows. And $\text{blkdiag}(\mathbf{X}, \mathbf{Y})$ is block diagonal operation from the matrixes \mathbf{X} and \mathbf{Y} .

II. SYSTEM MODEL

This paper considering a wireless MIMO (multi-input multi-output) broadcast system where one transmitter

equipped with N_T antennas transmits radio signals to one EH user with N_{EH} antennas and one ID user with N_{ID} antennas at the time and frequency as show in Fig.1.

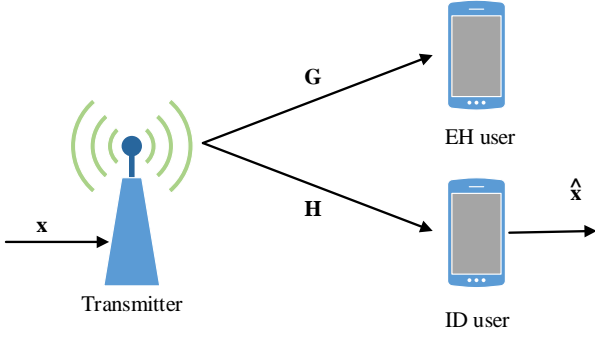


Fig.1 A MIMO broadcast system for simultaneous wireless information and power transfer.

Assuming a narrow-band transmission over quasi-static fading channels, the baseband equivalent channels from the transmitter to the EH receiver and ID receiver can be modelled by matrices $\mathbf{H} \in \mathbb{C}^{N_{ID} \times N_T}$ and $\mathbf{G} \in \mathbb{C}^{N_{EH} \times N_T}$,

respectively. It is assumed that at each fading state, \mathbf{G} and \mathbf{H} are both known at the transmitter, and separately known at the corresponding receiver. Note that for the case of collocated EH and ID receivers, \mathbf{G} is identical to \mathbf{H} and thus $N_{EH} = N_{ID}$.

Let $\mathbf{x} \in \mathbb{C}^{N_s \times 1}$ denotes the input signal vector where $\mathcal{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_{N_s}$ and N_s is the number of data-streams, the baseband transmission from the transmitter to the ID receiver can be modelled by

$$\mathbf{y} = \mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}, \quad (1)$$

where $\mathbf{F} \in \mathbb{C}^{N_T \times N_s}$ represents the linear precoder at transmitter, which is subject to the power constraint $\text{Tr}[\mathbf{F}\mathbf{F}^H] \leq P_T$ and $\mathbf{n} \in \mathbb{C}^{N_{ID} \times 1}$ denotes the receiver vector with $\mathcal{CN}(0, \mathbf{I}_{N_{ID}})$. Under these assumption, the recovered signal $\hat{\mathbf{x}} \in \mathbb{C}^{N_s \times 1}$ at the ID user can be modeled by

$$\hat{\mathbf{x}} = \mathbf{L}\mathbf{y} = \mathbf{L}(\mathbf{H}\mathbf{F}\mathbf{x} + \mathbf{n}), \quad (2)$$

where $\mathbf{L} \in \mathbb{C}^{N_{ID} \times N_{ID}}$ denotes the linear receiver post-coder at ID user.

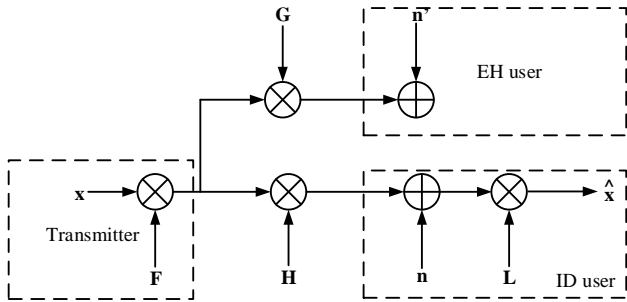


Fig. 2 Simple signal model for SWIPT.

III. PROBLEM FORMULATION

From (2), the MSE between the transmitted and recovered signals the ID user can be determined by

$$\text{MSE} = \text{Tr}((\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^H). \quad (3)$$

We consider the MIMO link from the transmitter to the EH receiver when the ID receiver is not present. In this case, harvested energy E at the EH can be expressed as

$$E = \delta \text{Tr}(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H), \quad (4)$$

where δ denotes a constant accounting for the harvesting efficiency and we assume $\delta = 1$ for simplicity unless stated otherwise.

With a given the precoding matrix \mathbf{F} for the transmit signal, the transmission rate from the transmitter to the information receiver can be written as

$$R = \log |\mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H|. \quad (5)$$

In this paper, precoding and post-coding matrices are derived to minimize MSE value at ID user. The problem of minimizing the MSE under the total transmit power constraint at ID user and the harvested energy constraint at the EH user can be formulated as

$$\min_{\mathbf{F}, \mathbf{L}} \text{Tr}((\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^H), \quad (6)$$

$$\text{s.t. } \text{Tr}[\mathbf{F}\mathbf{F}^H] \leq P_T, \quad (7)$$

$$\text{Tr}(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H) \geq \bar{E},$$

$$(8)$$

$$\mathbf{F}, \mathbf{L} \succeq 0 \quad (9)$$

where P_T denotes the maximum power constraint at the transmitter at ID user, \bar{E} is the minimum energy during the unit transmission time.

The above problem is non-convex due to the coupled beamforming matrices \mathbf{L} and \mathbf{F} . We divide this problem by two sub-problem as following:

A. Pre-coding matrix design

In this paper, precoding matrices are derived to maximize the sum-rate of information user. The problem of maximizing the total sum-rate under the total transmit power constraint at the transmitter can be formulated as

$$\text{P1: } \max R \quad (10)$$

$$\text{s.t. } \text{Tr}[\mathbf{F}\mathbf{F}^H] \leq P_T, \quad (11)$$

$$\text{Tr}(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H) \geq \bar{E}, \quad (12)$$

$$\mathbf{F} \succeq 0 \quad (13)$$

The associated Lagrange equation is expressed as

$$\mathcal{L}(\lambda, \mu, \mathbf{F}) = \log |\mathbf{I}_{N_s} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H| + \lambda(\text{Tr}[\mathbf{F}\mathbf{F}^H] - P_T) - \mu(\text{Tr}(\mathbf{F}^H\mathbf{G}^H\mathbf{G}\mathbf{F}) - \bar{E}) \quad (14)$$

where λ and μ are the non-negative dual variables associated with the constraints in (11) and (12), respectively.

The dual function $g(\lambda, \mu)$ is then defined as the optimal value of the following problem

$$g(\lambda, \mu) = \max_{\mathbf{F} \succeq 0} \mathcal{L}(\lambda, \mu, \mathbf{F}). \quad (15)$$

And the dual problem is defined as

$$\min_{\lambda > 0, \mu > 0} g(\lambda, \mu). \quad (16)$$

Since the optimization problem (P1) can be solved equivalently by solving (16), in the following, we first maximize the Lagrangian to obtain the dual function with fixed λ and μ , and then find the optimal dual solutions λ^* and μ^* to minimize the dual function. The precoding matrix \mathbf{F}^* that maximize the Lagrangian to obtain $g(\lambda^*, \mu^*)$ is thus the optimal solution of the original problem (P1).

Now, we consider the problem of maximizing the Lagrangian over \mathbf{F} with fixed λ and μ . This problem can be rewritten as

$$\max_{\mathbf{F} \succeq 0} \log |\mathbf{I}_{N_s} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H| + \text{Tr}((\mu^*\mathbf{I} - \lambda^*\mathbf{G}^H\mathbf{G})\mathbf{F}\mathbf{F}^H) \quad (17)$$

Let $\mathbf{P} = \mu\mathbf{I} - \lambda\mathbf{G}^H\mathbf{G}$ and $\mathbf{P} > 0$ [11]. The problem in (17) is then rewritten as

$$\max_{\mathbf{F} \succeq 0} \log |\mathbf{I}_{N_s} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H| + \text{Tr}(\mathbf{P}\mathbf{F}\mathbf{F}^H) \quad (18)$$

Define $\mathbf{Q} = \mathbf{P}^{1/2}\mathbf{F}$ and the above problem can be reformulated as

$$\max_{\mathbf{Q} \succeq 0} \log |\mathbf{I}_{N_S} + \mathbf{H}\mathbf{P}^{-\frac{1}{2}}\mathbf{Q}\mathbf{Q}^H\mathbf{P}^{-\frac{1}{2}}\mathbf{H}^H| + \text{Tr}(\mathbf{Q}\mathbf{Q}^H) \quad (19)$$

It has been shown in [11], the optimal solution to Problem (19) with arbitrary $\mathbf{P} > 0$ has the following form:

$$\mathbf{Q}^* = \mathbf{V}\mathbf{\Gamma}^{\frac{1}{2}} \quad (20)$$

Where \mathbf{V} is obtained from the SVD of $\mathbf{H}\mathbf{P}^{-\frac{1}{2}} = \mathbf{U}\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{V}^H$, with two unitary matrices $\mathbf{U} \in \mathbb{C}^{N_S \times M}$ and $\mathbf{V} \in \mathbb{C}^{N_T \times M}$, $M = \min(N_S, N_T)$, $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$, and $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$. $\mathbf{\Gamma} = \text{diag}(\gamma_1, \gamma_2, \dots, \gamma_M)$, where

$$\gamma_i = \max\left(1 - \frac{1}{\lambda_i}, 0\right), \quad i = 1, \dots, M \quad (21)$$

Thus, the optimal solution of the optimization problem (19) can be expressed as

$$\mathbf{F}^* = \mathbf{P}^{-\frac{1}{2}}\mathbf{V}\mathbf{\Gamma}^{\frac{1}{2}} \quad (22)$$

B. Post-coding matrix design

The optimization problem over post-coding matrix can be formulated as

$$\text{P2: } \min_{\mathbf{L}} \text{Tr}((\hat{\mathbf{x}} - \mathbf{x})(\hat{\mathbf{x}} - \mathbf{x})^H), \quad (23)$$

$$\text{s.t } \mathbf{L} \succeq 0 \quad (24)$$

The associated Lagrange equation is expressed as

$$\begin{aligned} \mathcal{L}(\mathbf{L}) &= \text{Tr}((\mathbf{L}\mathbf{H}\mathbf{F} - \mathbf{I}_{N_S})(\mathbf{L}\mathbf{H}\mathbf{F} - \mathbf{I}_{N_S})^H) + \sigma_n^2 \text{Tr}(\mathbf{L}\mathbf{L}^H) \\ &= \text{Tr}(\mathbf{L}\mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H\mathbf{L}^H - \mathbf{L}\mathbf{H}\mathbf{F} - \mathbf{F}^H\mathbf{H}^H\mathbf{L}^H + \mathbf{I}_{N_S}) + \sigma_n^2 \text{Tr}(\mathbf{L}\mathbf{L}^H) \end{aligned} \quad (25)$$

Taking the derivative $\mathcal{L}(\mathbf{L})$ with respect to \mathbf{L}^* and setting to zero, we get

$$\mathbf{L} = \mathbf{F}^H\mathbf{H}^H(\mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_S})^{-1} \quad (26)$$

C. Beamforming implementation algorithm

Based on the above pre/post-coding matrix designs, the implementation algorithm is demonstrated as Algorithm 1.

ALGORITHM 1: BEAMFORMING IMPLEMENTATION ALGORITHM

Init: $\lambda > 0, \mu > 0$
Repeat
Find the optimal \mathbf{F} according to (22)
Update λ and μ using ellipsoid method
Until converge to the prescribed accuracy
Find the optimal \mathbf{L} according to (26)

IV. SIMULATION RESULTS

In this section, we present numerical results to evaluate the performance of the proposed beamforming algorithm. The considered system parameters are summarized in Table I. Throughout our simulation, the channels are generated from Rayleigh fading pathloss model, $\mathbf{H} = d^{3/2}\tilde{\mathbf{H}}$ and $\mathbf{G} = d^{3/2}\tilde{\mathbf{G}}$ where each element of $\tilde{\mathbf{H}}$ and $\tilde{\mathbf{G}}$ is generated independently with i.i.d. complex proper zero-mean Gaussian components with variance 1. We report the average results for 100 channel realizations.

By comparing the Rate - Energy tradeoff regions in Figure 3, one can also confirm that our solution maintains optimality for all target energy levels. The Rate-Energy (R-E) region, which consists all the achievable rate and the harvested energy pair for a given power constraint, is defined as [10]

$$\mathcal{C}_{R-E}(P_T) = \{(R, E): R \leq \log|\mathbf{I} + \mathbf{H}\mathbf{F}\mathbf{F}^H\mathbf{H}^H|, E \leq \text{Tr}(\mathbf{G}\mathbf{F}\mathbf{F}^H\mathbf{G}^H), \text{Tr}(\mathbf{F}\mathbf{F}^H) \leq P_T\}. \quad (27)$$

As show in this figure, the information rates reduce as harvested energy increases.

TABLE I
SIMULATION SYSTEM SETTINGS.

Number of antennas transmitter	$N_T=4$
Number of antennas EH receivers	$N_{EH}=4$
Number of antennas ID receivers	$N_{ID}=4$
Number of data-streams	$N_S=4$
Noise Power Spectral Density	-100dBm/Hz
Energy Conversion Efficiency	50% ($\delta=0.5$)
Pathloss Exponent	3
Signal Bandwidth	10 MHz
Distance: Tx - ID receiver	$d_h = 10(m)$
Distance: Tx - EH receiver	$d_g = 10(m)$

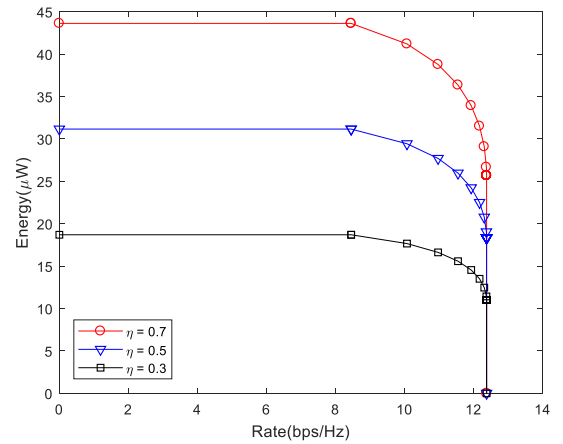


Fig. 3. Rate-Energy tradeoff regions for MIMO system, $P=10\text{dBm}$, $N_T = N_{EH} = N_S = 4$.

Figure 4 shows the information rate performances as a function of receive signal-to-noise-ratio (SNR). The harvested energy constraint is set to be haft of the maximum energy that can be obtained. When the number of antenna increases, the information rate also increases.

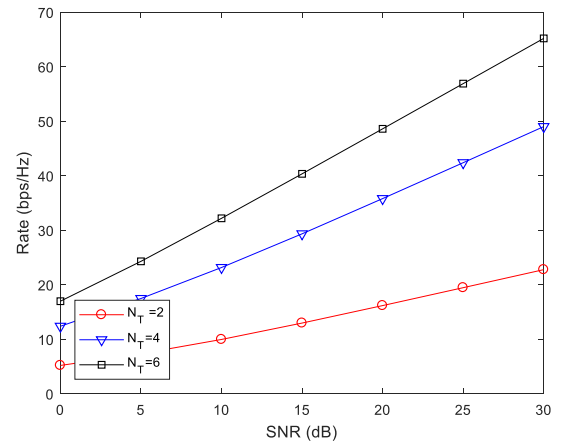


Fig. 4. The information rate performances as a function of receive signal-to-noise-ratio (SNR).

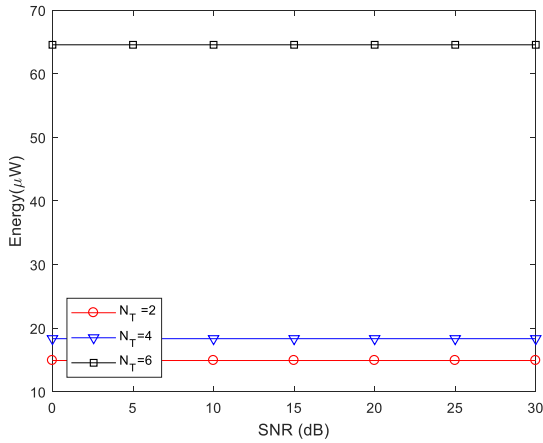


Fig.5. Harvested energy performance comparison in $P = 10dBm$, $N_r = N_{EH} = N_s$.

Fig. 5 depicts the harvested energy in SNR, from the figure, which shows that the higher the harvested energy, the greater the number of antennas.

V. CONCLUSION

This paper proposed the beamforming algorithms for MIMO system with SWIPT. Firstly, a maximize the information rate algorithm is introduced, and the minimize the MSE information received is presented. Finally, we have provided the numerical results to show the performance of the proposed algorithm.

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